Name Key

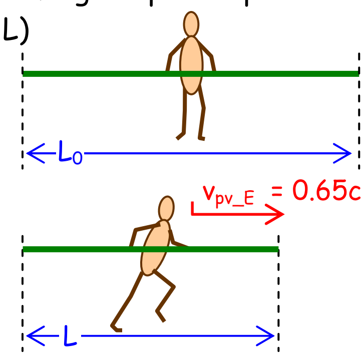
Quiz 10

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}},\quad \Delta t = \gamma \Delta t_0,\quad L = \frac{L_0}{\gamma}$$

1) A pole vaulter at the Relativistic Olympics sprints past you with a speed of 0.65 c. When he is at rest, his pole is 7.0 m long.

a)[2] What is gamma?

b)[2] What length do you perceive the pole to be as he passes you, assuming his pole is parallel to his motion? (Draw the poles for him at rest and running & label L_0 & L)



$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{0.65^2c^2}{c^2}}} = \frac{1}{\sqrt{1-0.4225}} = 1.32$$

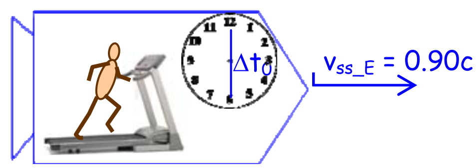
$$L = \frac{L_0}{\gamma} = \frac{7\text{m}}{1.32} = 5.32\ \text{m}$$

The proper length occurs in the reference frame where the object (or distance) is at rest. The pole is at rest in the runner's reference frame.

2) A spacecraft moves past a student with a relative velocity of 0.90 c. The pilot of the spacecraft works out for 30 minutes on her watch.

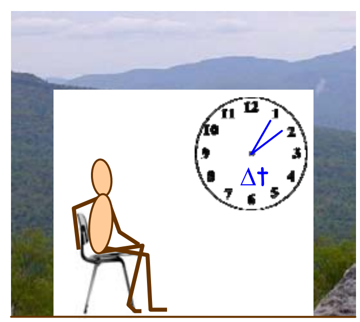
a)[2] What is gamma?

b)[2] How long does the pilot exercise according to the student? (Draw hands on Earth's clock and label Δt_0 & Δt)



$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{0.90^2c^2}{c^2}}} = \frac{1}{\sqrt{0.19}} = 2.29$$

$$\Delta t = \gamma\Delta t_0 = 2.29(30\,\text{min}) = 68.8\ \text{min} = 1\ \text{hr}\ 8.8\ \text{min}$$



The proper time is in the reference frame where the clock stays in the same place — the pilot's clock is in the same place for her at the beginning and end of her exercises.

3)[2] Solve the Lorentz factor for v: $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

Square both sides: $\gamma^2 = \frac{1}{1-\frac{v^2}{c^2}}$

Multiply both sides by the denominator: $\gamma^2\left(1-\frac{v^2}{c^2}\right) = 1$

Divide by γ^2: $1-\frac{v^2}{c^2} = \frac{1}{\gamma^2}$

Move the 1: $\frac{v^2}{c^2} = 1-\frac{1}{\gamma^2}$

Multiply by c^2: $v^2 = \left(1-\frac{1}{\gamma^2}\right)c^2$

Take the square root of both sides: $v = \sqrt{1-\frac{1}{\gamma^2}}\,c$